

# Inverse Method for the Design of Multielement High-Lift Systems

T.D. Beatty\* and J.C. Narramore†

*Douglas Aircraft Company, McDonnell Douglas Corporation, Long Beach, Calif.*

A multielement inverse potential flow program has been developed which will determine the section geometry required to produce a desired upper-surface pressure distribution on one or more airfoils of a two-dimensional multielement airfoil system. An iterative procedure is used which modifies the camberlines of bodies on which the pressure is specified so that the calculated upper-surface pressure distributions approach the desired pressure distributions. Two options are available: one that holds a constant thickness while changing the camberline, and one that holds the lower surface shape constant, while changing the camberline and thickness. Both options require that the chord lengths and relative positions are held constant during the iterative procedure. Examples of the results of the inverse process are presented which illustrate the accuracy of the program and its potential application to the design of improved high-lift systems.

## Introduction

**A**ERODYNAMIC design problems become more complex as demands on aircraft performance and efficiency increase. Therefore, development of accurate analytical methods that can be utilized by the aerodynamic design engineer to produce the most efficient vehicle is very important. Historically, subsonic airfoil design was afforded by the use of direct inviscid flowfield solutions requiring many iterations of geometric parameters and consuming much of the designers' time with tedious tasks. The obtained solutions were analyzed carefully before final wind-tunnel testing was begun. After the results of the wind-tunnel tests were compiled, further alterations to the configuration might still have to be made to achieve the desired characteristics.

Inverse airfoil design computer programs recently have accounted for a change in the philosophy associated with the design procedure. Advantages of the inverse methods stem from the fact that they allow the designer to specify the pressure distribution and thereby the lift, pitching moment, and boundary-layer development characteristics of the configuration. Therefore, the performance of the system is specified, and the inverse solution gives the geometry that will produce that performance.

Two main categories, exact-transformation and iterative-direct, encompass all inverse airfoil design methods. Analytically exact solutions characterize methods falling in the exact-transformation category, making these solutions very valuable. However, currently operational transformation methods are limited to a single airfoil. Although encouraging work on a two-element exact-transformation inverse has been done, there does not seem to be much hope for extending this type of method to the case of three-or-more-element airfoils.<sup>1</sup>

Thus, at the present time, multielement inverse design methods<sup>2-4</sup> rely on the utilization of iterative-direct techniques. These methods use a direct solution that is not as sophisticated as the higher-order Douglas-Neumann, and they are not formulated with constraints that would make them applicable to the practical high-lift system design problem.

Submitted June 9, 1975; presented as Paper 75-879 at the AIAA 8th Fluid and Plasma Dynamics Conference, Hartford, Conn., June 16-18, 1975; revision received Sept. 8, 1975. This paper summarizes work performed under the sponsorship of the Independent Research and Development Program (IRAD) of the McDonnell Douglas Corp.

Index categories: Aircraft Aerodynamics (including Component Aerodynamics); Subsonic and Transonic Flow.

\*Engineer/Scientist-Specialist, High-Lift Technology Development, Aerodynamics Subdivision.

†Engineer/Scientist; presently at Cessna Aircraft Company.

In an iterative-direct method, an accurate direct potential flow method is coupled with an inverse algorithm, which will modify a specified initial geometry automatically in an attempt to achieve a required pressure distribution. In this approach, for realistic flows, the iterative-direct scheme will reduce the difference between the calculated and required pressure distributions to an arbitrarily small amount, depending upon the number of iterations.

A well-designed direct flowfield method always gives a solution, because for every geometry there is a corresponding pressure distribution. However, the same statement is decidedly not true for an inverse method. In general, an arbitrarily selected pressure distribution does not correspond to a physically meaningful geometry. Thus, the true function of an inverse method of any type is to find the body whose pressure distribution is closest in some sense to the prescribed distribution. Often the prescribed pressure distribution must be altered by the designer, who thus remains in the "loop," but to a lesser extent than is required if only direct calculation methods are used. Through wind-tunnel tests, the actual performance of two-dimensional single-element airfoil sections designed utilizing inverse computer programs has been shown to correlate with predicted design values.<sup>5</sup> The purpose of this paper is to present an iterative-direct inverse computer program developed as a design tool for multielement, high-lift configuration airfoil design.

## Discussion

In an iterative-direct inverse procedure, a method must be devised to find the correction to the body which will reduce the difference between the pressure distribution desired and the pressure that is calculated by the direct solution for the current configuration. Wilkinson's method<sup>2</sup> determines the vorticity distribution along the mean line that is necessary to remove this velocity difference. Normal velocities are induced at the mean line because of the vorticity distribution, implying that the mean-line slope should be changed so that it is a streamline in the flow resulting from the combination of the normal induced velocity and the current flowfield. The thickness distribution of the body is not changed.

Wilkinson's method gives satisfactory results for some test cases even though it uses a fairly simple potential flow technique. It was felt that Wilkinson's approach, used with a more general and more accurate potential flow calculation routine, such as the Douglas-Neumann potential flow method,<sup>6</sup> having parabolic surface elements with linear singularity variation, would yield a much more versatile

method capable of obtaining a higher degree of accuracy with the same number of elements. The greater accuracy of the Douglas-Neumann method as compared to Wilkinson's approach is shown in Fig. 1, which presents a comparison of the computed pressure distributions of Wilkinson and Douglas-Neumann with the exact solution for a large camber Karman-Trefftz airfoil. Because of the increased accuracy shown in the preceding, an inverse procedure based on the Douglas-Neumann method has been developed that is a close analogy, with generalizations, to the original Wilkinson approach.

Since the present approach was developed for multielement high-lift systems in which some of the geometries are required to be fixed, it is necessary for the method to be capable of analyzing configurations where the pressure is specified on one or more elements and the geometry is specified on the remaining elements. Also, since the capability of calculating the effects of wind-tunnel walls on any configuration is desired, the method must be able to calculate the flow about nonlifting bodies. Accordingly the following types of geometries are allowed in the present approach: 1) nonlifting bodies such as wind-tunnel walls, 2) lifting airfoils with geometry specified, and 3) lifting airfoils with pressure specified. During the course of the iterations, the nonlifting geometries remain fixed, while the lifting airfoils move in such a way that the relative location of the leading edge of each airfoil remains fixed with respect to the trailing edge of the airfoil immediately upstream. All airfoil chords are preserved. The airfoils on which pressures are not specified change only location and do not change shape whereas the airfoils on which pressure is specified, of course, change shape. Figure 2 depicts the solution procedure which performs the iterative-direct operation automatically.

The first step encountered in the present inverse method is the calculation of thickness distributions and initial mean-line shapes for the pressure-specified bodies. It should be noted that the input for the Douglas-Neumann method consists of points on each airfoil surface. Determination of the mean line and thickness from this type of data may be accomplished by determining the centers of circles that are tangent to the upper and lower surfaces of the sections.

The next step is to determine the vorticity distribution on the mean line which will cancel the difference between the specified and the calculated pressures on the upper surface. Normal and tangential induced velocity matrices must, therefore, be calculated for the mean line and upper surface of all of the pressure-specified bodies, using routines found in the Douglas-Neumann Potential Flow Program. After a direct solution has been obtained, a calculated velocity array exists which can be compared to the desired velocity array to determine the velocity difference array. Knowing the velocity difference array  $\Delta V_i$  and the tangential induced velocity matrix  $B_{ij}^* \gamma$ , the vorticity distribution  $\gamma$  on the mean line that will cancel the velocity difference array may be determined from

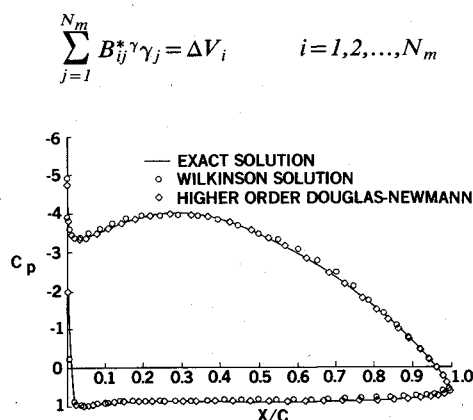


Fig. 1 Comparison of pressure distributions calculated by two potential flow methods with the exact solution for a large camber Karman Trefftz airfoil.

where  $N_m$  is the total number of mean-line elements. The tangential induced velocity matrix is formed by computing the tangential velocities on the upper surface due to vorticity distributions on the mean lines. In order to avoid numerical difficulty, and exception is made when computing induced tangential velocity on the upper surface of the body due to an element of vorticity on its own mean line. In this case, tangential velocities are evaluated at the midpoints of elements on the mean line and projected to the upper surface by multiplying them by the cosine of the angle between the local tangent to the mean line and the local tangent to the upper surface (the asterisk is to call attention to this fact).

By use of the normal induced velocity matrix ( $A_{ij}$ ), velocities normal to the mean line  $V_{ni}$  are computed from

$$V_{ni} = \sum_{j=1}^{N_m} A_{ij} \gamma_j$$

The velocities are induced by vorticity elements. However, they are computed within the Douglas-Neumann routines from the corresponding source velocities by interchanging components and making one component negative, since

$$A_{ij} \gamma_j = -B_{ij}$$

$$B_{ij} \gamma_j = A_{ij}$$

where  $A_{ij}$  and  $B_{ij}$  are, respectively, the normal and tangential induced velocity matrices due to a source distribution, and  $A_{ij}^*$  and  $B_{ij}^*$  are the normal and tangential velocity matrices due to a vorticity distribution. A detailed explanation of the calculation of the induced velocities and the capabilities of the Douglas-Neumann method are presented in Refs. 7 and 8.

Calculated normal velocities are interpreted as a required change in slope of the mean-line element by means of the approximate relation

$$\Delta n_i / \Delta s_i = V_{ni}$$

where  $\Delta n_i$  is the local movement of the mean-line end point normal to the mean line, and  $\Delta s_i$  is the length of the mean-line element. For simplicity this relation ignores the difference between local tangential velocities and the freestream velocity.

New mean lines are determined by integrating the slope changes. The original thickness then is wrapped around the new camberline to determine the new configuration. The whole process is iterated until the desired solution is obtained. This inverse algorithm may be used to determine the shape of multielement airfoil sections, which will produce a given upper-surface pressure distribution by use of the iterative-direct technique.

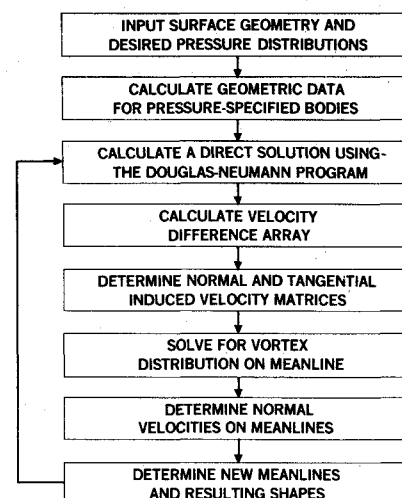


Fig. 2 Flow chart depicting the solution procedure.

### Extension of the Method

Seldom in the design of high-lift systems for aircraft is the freedom to modify the total geometry allowed. At the present state-of-the-art, the high-lift system configuration designer usually is given freedom to modify only those parts of the segmented airfoil not dictated by high-speed cruise considerations (Fig. 3).

It was desirable, therefore, to develop a new inverse method capable of calculating a modification to only part of the upper surface of an airfoil to produce a satisfactory pressure distribution on the section. Accordingly, an extension of the basic inverse method has been developed that will perform the mixed boundary condition inverse operation, i.e., an inverse solution can be determined which allows a prescribed region on the upper surface to be modified while the lower surface and the region aft of the prescribed area on the upper surface remain unchanged.

The solution method for the mixed boundary condition inverse is, again, an iterative process with the following steps required to calculate a new configuration: 1) calculate a direct solution, using the Douglas-Neumann program to determine the pressure distribution on the current configuration; 2) compare the calculated pressure to the desired pressure in the pressure specified region of the upper surface; 3) determine the vorticity distribution on the mean line that will produce the required change in the velocity on the upper surface in the pressure specified region; 4) compute the normal velocity at the mean line due to this vorticity distribution; 5) interpret the normal velocity as a change in the slope of the mean line, with the constraint that the last mean-line point with pressure specified may not move; 6) calculate the new upper surface due to the change in the mean line without changing the lower surface; and 7) determine the new thickness distribution and mean line.

It is seen easily that this method uses the same solution procedures as the basic program. However, the thickness distribution is allowed to change in the pressure specified

region. This option will provide a practical tool to the aerodynamic design engineer for the design of improved high-lift systems.

### Test Cases

In order to establish the validity of the multielement inverse potential flow program, calculations for a series of test cases were performed using both inverse techniques. First, so that the accuracy of the inverse procedure could be determined, test cases were devised which consisted of obtaining a solution for a configuration by means of a direct solution, and then, starting from a different configuration, determining how well the inverse procedure matches the direct solution. During the direct solution, the thickness distributions and pressure distributions on the bodies were found. These thickness distributions then were used to form new bodies with different camberlines for input to the inverse program. The specified pressures were the ones determined in the direct solution. Comparisons of the results of the inverse solution and the direct solution then can be used to determine the accuracy of the inverse program.

The first case chosen for correlation was a single-element airfoil which, at an angle of attack of  $14^\circ$ , has a large leading edge velocity spike as computed in the direct solution by the Douglas-Neumann program. The thickness distribution was used as the input geometry for the inverse program. The pressure distribution calculated in the direct solution by the Douglas-Neumann program was used as the desired pressure distribution, which also was input. Figure 4 shows that the inverse program not only computed the airfoil geometry and the pressure recovery distribution accurately but also produced the velocity peak at the nose.

Results for a supercritical single-element airfoil are shown in Fig. 5. Comparison of the pressure distribution and geometry of the input configuration (illustrated by a dashed line) and the desired results (solid lines) shows that the aft section of the airfoil must be cambered greatly to produce the high loading in this region. Five inverse iterations produced the solution of Fig. 5. Figure 6 shows that the small discrepancies in the desired and resulting pressure distribution of Fig. 5

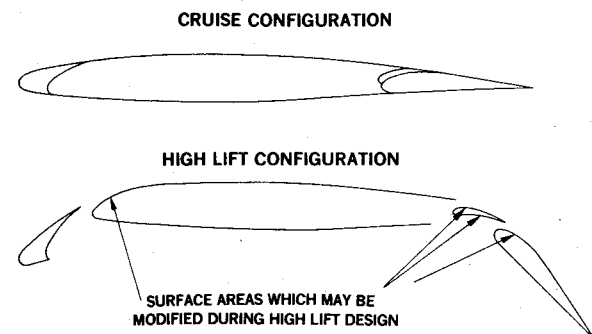


Fig. 3 Surfaces which may be modified by the high-lift designer.

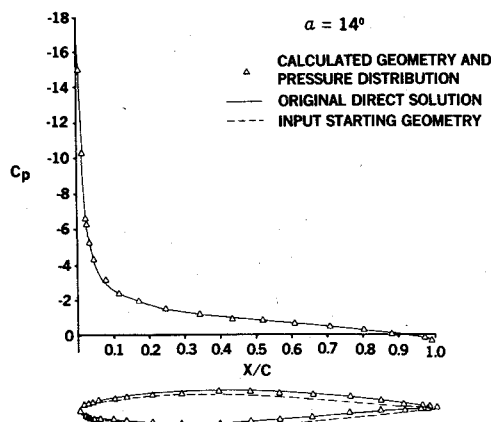


Fig. 4 Single-element inverse verification case.

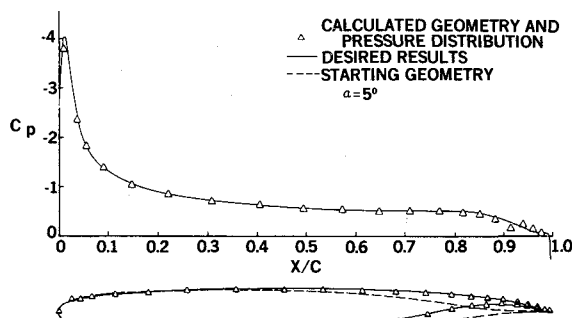


Fig. 5 Fifth iteration of supercritical airfoil inverse.

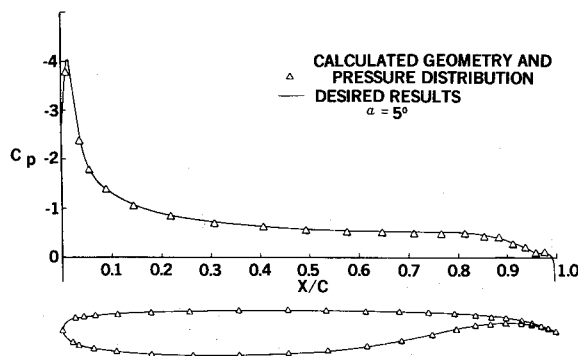


Fig. 6 Fifteenth iteration of supercritical airfoil inverse.

can be eliminated after 15 iterations. Five iterations for this case required 0.246 min of CPU time on an IBM 370/168.

The desirability of this program is based on the fact that it is designed to perform the inverse operation on multielement configurations and to take into account the interactive influences of multielement airfoils. An input configuration for a two-element system is presented in Fig. 7. It is shown that the pressures on the upper surfaces of both elements must be lowered to achieve the desired pressure distributions. Figure 8 shows that the desired pressure distribution and geometry were achieved after five iterations. Each iteration of this case required approximately 0.053 min of CPU time on an IBM 370/168. It is evident in the figure that the inverse procedure not only produced a camber change in the flap, but also reproduced the proper angle with respect to the main section.

The three-element inverse capability, which is unique to iterative-direct inverse methods, is demonstrated in Fig. 9. Figure 9a shows the input geometry consisting of symmetrical airfoils with the same thickness distributions as the desired airfoils and the pressure distribution on the input configuration. Comparing the desired and calculated pressures of Fig. 9a, it can be seen that large loading changes are required on all of the sections. Results of the fifth iteration are compared to the desired solution in Fig. 9b. Notice that the pressures and geometries are reproduced accurately, and the dove-tail region of the main airfoil is determined correctly.

Figure 10 illustrates the ability of the inverse program to handle the mixed problem where the shapes of some sections are specified and the pressure distributions on other sections are defined. In the case shown, the airfoil geometry is given for the first two elements and the pressure is given for the third element. After five iterations, the inverse solution for the geometry and pressure distribution of the flap approached

the desired results. This capability to modify selected elements of a multielement airfoil system is very useful in the practical design of high-lift systems.

The preceding examples have shown the ability of the present method to obtain an inverse solution with little or no discrepancy between specified and obtained pressure distributions for a wide variety of cases. In addition to these examples, cases were run in which the pressure distributions on airfoils were modified without prior knowledge of the correct solution. Also, it should be mentioned that all of the following cases represent the first solution obtained. No perturbations of the input parameters were performed so that a better agreement between desired and calculated pressures might be found.

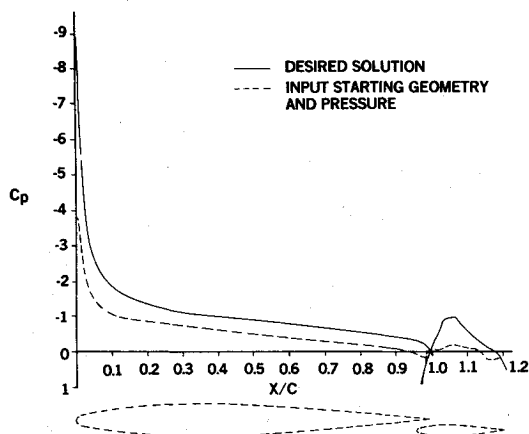


Fig. 7 Input configuration for two-element inverse.

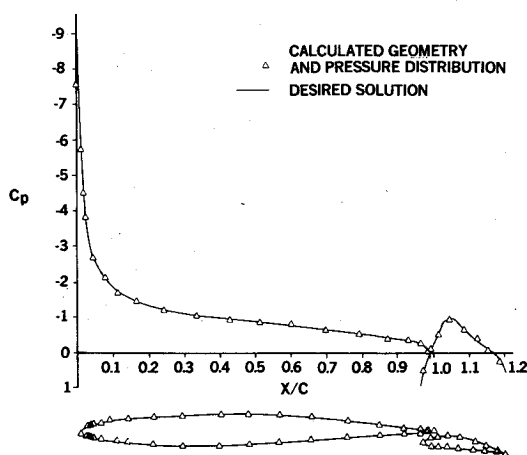


Fig. 8 Fifth iteration of two-element inverse.

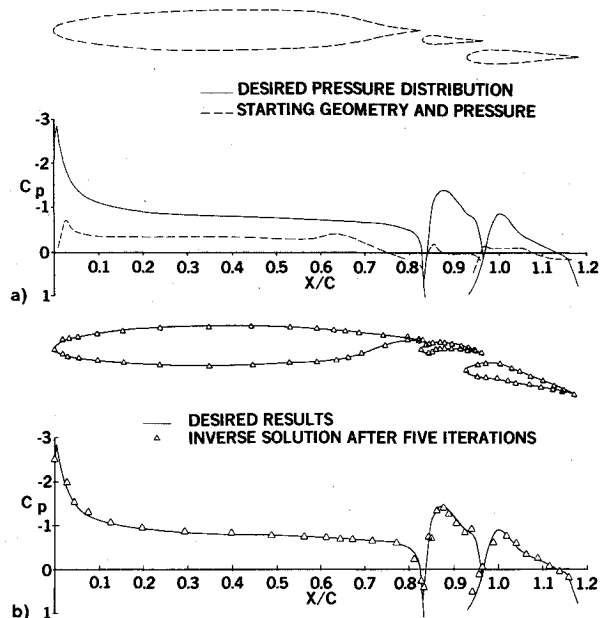


Fig. 9 a) Input geometry for three-element inverse; b) fifth iteration of three-element inverse.

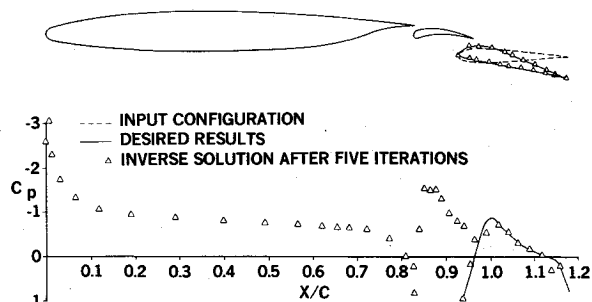


Fig. 10 Inverse solution for three-element case with one element having its pressure specified.

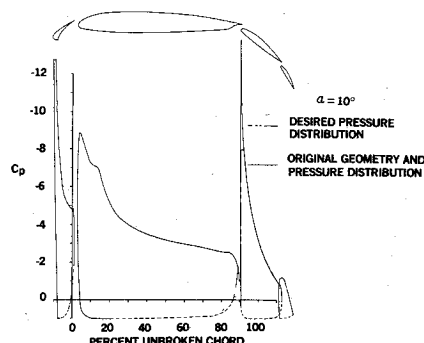


Fig. 11 Input configuration of flap design for a 12% thick airfoil.

The first case considered was a flap system design for a 12%-thick airfoil. The original geometry, shown in Fig. 11, produced a large velocity peak at the front of the flap which the boundary layer cannot withstand without separation. The inverse routine was applied to determine whether a new flap shape could be defined which would lower that peak. Figure 12 shows the results of the inverse operation after five iterations. The peak on the flap was indeed reduced, but the pressure produced was not exactly the input-desired pressure distribution, indicating that the specified pressure that was chosen arbitrarily will not produce a realistic geometry.

Figure 13 shows an attempt to design a slat that is to maintain a high loading over a considerable chordwise extent. The input configuration shown in Fig. 13a has a velocity spike at the nose of the slat which produces a minimum pressure coefficient of  $-30.01$ . The objective was to hold a minimum pressure of  $-8.5$  on the slat and preserve the geometry of the wing section. Figure 13b shows the resulting shape and pressures as computed by the inverse technique. Notice how the slat is modified into a shape which is highly cambered in order to produce a distribution that has a constant load region.

Another case that was examined was a two-element airfoil designed for high lift without boundary-layer separation. A stratford-type pressure distribution, which allows complete recovery without separation, was specified on two equal-

chord Liebeck-type elements.<sup>9</sup> Because of the circulation effect, the aft element induces an increase in circulation on the forward body.<sup>10</sup> Therefore, the forward element was arbitrarily assigned a velocity ratio-squared ( $V$ )<sup>2</sup> of twice that of the aft element. The input configuration is shown in Fig. 14. As can be seen in Fig. 15, the bodies were considerably modified and the pressure distribution approached the desired pressure distribution after the inverse procedure had been applied. Again, modifications of the desired pressure distributions (such as raising the front element velocity/squared ratio to 3 times that of the aft element, etc.) could lead to a closer agreement between desired and obtained pressures.

An example of using the upper-surface inverse option is presented in Fig. 16. For this example, the flap system design for a 12%-thick airfoil was utilized as the original configuration (see Fig. 11). In this case, the velocity peak was to be limited to a lower value by only modifying part of the upper surface of the flap which constitutes a practical design

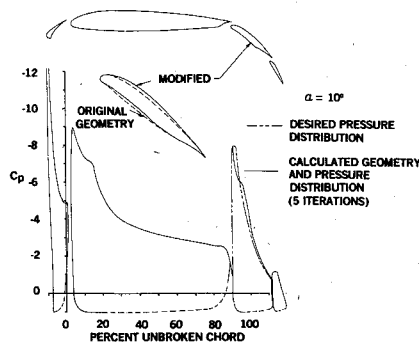


Fig. 12 Solution given by inverse operation after fifth iteration on flap design for 12% thick airfoil.

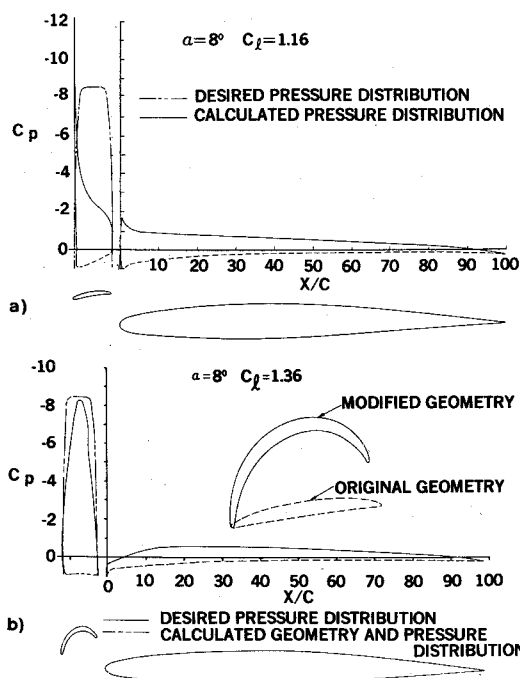


Fig. 13 a) Input geometry for high load slat design; b) inverse solution for high load slat design.

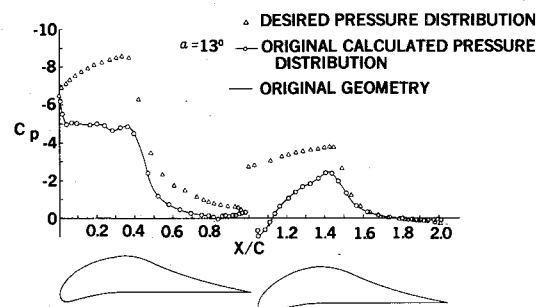


Fig. 14 Input configuration for two-element airfoil designed for high lift without separation.

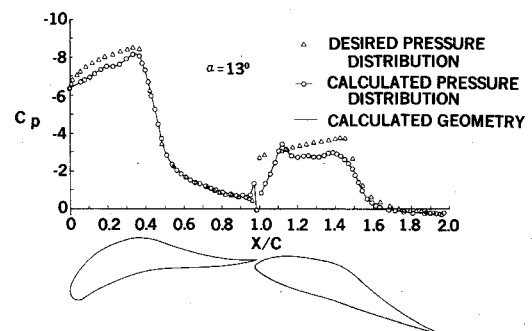


Fig. 15 Solution produced by inverse program for two-element airfoil designed for high lift without separation.

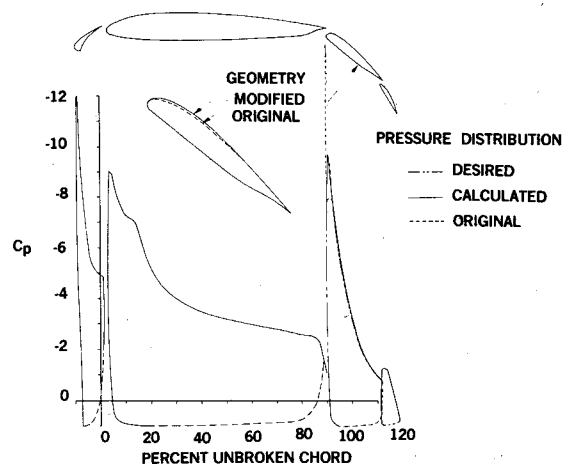


Fig. 16 Upper surface inverse option checkout case on flap design for a 12%-thick airfoil.

problem constraint. Figure 16 shows that the inverse routine lowered the minimum pressure peak but did not lower it to the desired level, since that solution is not physically possible. If the designer found this solution still undesirable further modifications of the desired pressure distribution would lead to different solutions.

### Conclusions

The availability of an inverse multielement airfoil design method can provide considerable insight into potential aerodynamic improvement for high-lift systems. Questions concerning the optimization of high-lift configurations now may be approached by other than trial-and-error methods, with an attendant reduction in the time required.

A method has been described for calculating the section shapes required to produce arbitrarily specified pressure distributions on the upper surfaces of one or more airfoils. The theoretical formulation of the potential flow method utilized is exact and accounts for all interference effects between elements. The inverse method is an iterative-direct procedure, which modifies the mean line of pressure-specified bodies, while the chord length, relative positions, and thickness or lower surface are held constant.

### References

- <sup>1</sup>James, R.M., "Analytical Studies of Two-Element Airfoil Systems," McDonnell Douglas Co., Long Beach, Calif., Rept. No. MDC J5831, Feb. 1974; prepared for Office of Naval Research under Contract No. N00014-72-C-0218.
- <sup>2</sup>Wilkinson, D.H., "A Numerical Solution of the Analysis and Design Problems for the Flow Past One or More Aerofoils or Cascades," Aeronautical Research Council, London, 3545, 1967.
- <sup>3</sup>Chen, A.W.-S., "The Determination of the Geometries of Multiple-Element Airfoils Optimized for Maximum Lift Coefficient," NASA-TM-X-67591, 1971.
- <sup>4</sup>Mavriplis, F., "Aerodynamic Prediction and Design Methods of Aircraft High-Lift Systems," Proceedings of NRC/ACOA Seminar on Aerodynamics, Ottawa, Canada, March 1975.
- <sup>5</sup>Liebeck, R.H., "A Class of Airfoils Designed for High Lift in Incompressible Flow," *Journal of Aircraft*, Vol. 10, Oct. 1973, pp. 610-617.
- <sup>6</sup>Hess, J.L. and Smith, A.M.O., "Calculation of Potential Flow about Arbitrary Bodies," *Progress in the Aeronautical Sciences*, Vol. 8, Pergamon Press, New York, 1966.
- <sup>7</sup>Hess, J.L., "Higher Order Numerical Solution of the Integral Equation for the Two-Dimensional Neuman Problem," *Computer Methods in Applied Mechanics and Engineering*, Vol. 2, 1973, pp. 1-15.
- <sup>8</sup>Hess, J.L., "Review of Integral-Equation Techniques for Solving Potential-Flow Problems with Emphasis on the Surface-Source Method," *Computer Methods in Applied Mechanics and Engineering*, Vol. 5, 1975, pp. 145-196.
- <sup>9</sup>Stratford, B.S., "The Prediction of Separation of the Turbulent Boundary Layer," *Journal of Fluid Mechanics*, Vol. 5, 1959.
- <sup>10</sup>Smith, A.M.O., "Aerodynamics of High-Lift Airfoil Systems," *Fluid Dynamics of Aircraft Stalling*, AGARD CP102, 1972.

*From the AIAA Progress in Astronautics and Aeronautics Series . . .*

## AEROACOUSTICS: FAN, STOL, AND BOUNDARY LAYER NOISE; SONIC BOOM; AEROACOUSTIC INSTRUMENTATION—v. 38

*Edited by Henry T. Nagamatsu, General Electric Research and Development Center; Jack V. O'Keefe, The Boeing Company; and Ira R. Schwartz, NASA Ames Development Center*

*A companion to Aeroacoustics: Jet and Combustion Noise; Duct Acoustics, volume 37 in the series.*

Twenty-nine papers, with summaries of panel discussions, comprise this volume, covering fan noise, STOL and rotor noise, acoustics of boundary layers and structural response, broadband noise generation, airfoil-wake interactions, blade spacing, supersonic fans, and inlet geometry. Studies of STOL and rotor noise cover mechanisms and prediction, suppression, spectral trends, and an engine-over-the-wing concept. Structural phenomena include panel response, high-temperature fatigue, and reentry vehicle loads, and boundary layer studies examine attached and separated turbulent pressure fluctuations, supersonic and hypersonic.

Sonic boom studies examine high-altitude overpressure, space shuttle boom, a low-boom supersonic transport, shock wave distortion, nonlinear acoustics, and far-field effects. Instrumentation includes directional microphone, jet flow source location, various sensors, shear flow measurement, laser velocimeters, and comparisons of wind tunnel and flight test data.

509 pp. 6 x 9, illus. \$19.00 Mem. \$30.00 List

TO ORDER WRITE: Publications Dept., AIAA, 1290 Avenue of the Americas, New York, N. Y. 10019